Building energy simulation > what it predicts

EXTERNAL ENERGY SOURCES

ENERGY STORAGE

THERMAL ZONE

INTERNAL ENERGY SOURCES

ENERGY INPUT

ENERGY INPUT BY AIR/WATER HVAC SYSTEMS

ENERGY INPUT

ENERGY INPUT

WATER HVAC SYSTEMS
What is (computational) simulation?

- From latin “simulare” – to pretend
- Using a mathematical model of a system to predict its output for a given input
- Models can be of many different types
- Models can be for whole systems or for subsystems
- Models depend on their function (desired output)
- Models always have limited validity range
(Computational) modeling approaches

- Deterministic or stochastic
- Time or event driven
- Continuous or discrete time
- Steady state or time dependent
- Static or dynamic
- White, grey or black box
- Special purpose or integrative
- Simple or detailed
- ....
Building energy simulation > how it works

- **SOLAR MODELS**
  - Solar Energy Inputs
  - Shading Geometry
  - Solar Energy Inputs

- **GLAZING MODELS**
  - Heat Input
  - Weather Data

- **INTERNAL SOURCES MODELS**
  - Heat Input
  - Time History

- **BUILDING FABRIC MODELS**
  - Heat Input
  - Weather Data
  - Materials Properties

- **OVERALL AIR HEAT BALANCE MODEL**
  - Heat Input
  - Time History

- **HVAC SYSTEMS**
  - Heat Input
Building energy simulation > inputs & outputs

**INPUTS**
- Weather data
- Building geometry
- Construction type
- HVAC type / usage
- Occupancy info
  - Quantity of users
  - Lights
  - Equipment
  - Usage

**OUTPUTS**
- Space temperatures
- Surface temperatures
- Humidity levels
- HVAC parameters
- Energy consumption
  - Component
  - System
  - Whole-building
Building energy flow paths

**Everything is in balance and influences other things**

\[
C_z \frac{dT_z}{dt} = \sum_{i=1}^{N_{si}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i (T_{si} - T_z) + \sum_{i=1}^{N_{zones}} m_i c_p (T_{zi} - T_z) + \dot{m}_{inf} c_p (T_{inf} - T_z) + \dot{Q}_{sys}
\]
Building energy modeling techniques

- Steady-state
  - No accurate inclusion of many effects
  - No dynamic response of buildings
  - Limited use in design stage (accuracy!)
Building energy modeling techniques

- **Steady-state**
- **Simple dynamic**

- Based on regression techniques
- Original results from more powerful modelling systems
- Results in tabular, graphical format
Building energy modeling techniques

- Steady-state
- Simple dynamic
- Numerical

- Approximation of partial differential equations
- Spatial/temporal integrity
- Handle complex flow paths
- Time varying parameters
- Handle systems with different time constants.
Building energy modeling techniques

Mathematical treatment determines the accuracy and flexibility

Building energy modeling details (example)

- **Surface convection**
  - Time-varying, surface averaged (correlations)
  - External: wind speed, direction
  - Internal: - characteristics
    - modelling convection (air flow)
Building energy modeling details (example)

Table 1  
$h_c$ correlations for mixed flow*

<table>
<thead>
<tr>
<th>Surface</th>
<th>$h_c$ correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls</td>
<td>$\left( \left{ \left[ 1.5 \frac{(AT)^{1/4}}{D_{h}} \right]^{6} + \left[ 1.23 \Delta T^{1/3} \right]^{6} \right}^{(3x1/6)} + \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [-0.199 + 0.190 \times (ac/h)^{0.8}] \right}^{3} \right)^{1/3}$</td>
</tr>
<tr>
<td>Assisting forces</td>
<td>Max of $\left[ \left[ 1.5 \frac{(AT)^{1/4}}{D_{h}} \right]^{6} + \left[ 1.23 \Delta T^{1/3} \right]^{6} \right]^{3.1/6}$ and $\left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [-0.199 + 0.190 \times (ac/h)^{0.8}] \right}$</td>
</tr>
<tr>
<td>Opposing forces</td>
<td>$80% \left[ \left[ 1.5 \frac{(AT)^{1/4}}{D_{h}} \right]^{6} + \left[ 1.23 \Delta T^{1/3} \right]^{6} \right]^{1/6}$</td>
</tr>
<tr>
<td>Floor</td>
<td>$80% \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [-0.199 + 0.190 \times (ac/h)^{0.8}] \right}$</td>
</tr>
<tr>
<td>Buoyant</td>
<td>$\left{ \left[ 1.4 \frac{(AT)^{1/4}}{D_{h}} \right]^{6} + \left[ 1.63 \Delta T^{1/3} \right]^{6} \right}^{(3x1/6)} + \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [0.159 + 0.116 \times (ac/h)^{0.8}] \right}^{3}$</td>
</tr>
<tr>
<td>Stably stratified</td>
<td>$\left[ 0.6 \frac{(AT)^{1/4}}{D_{h}} \right]^{3} + \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [0.159 + 0.116 \times (ac/h)^{0.8}] \right}^{3}$</td>
</tr>
<tr>
<td>Ceiling</td>
<td>$\left{ \left[ 1.4 \frac{(AT)^{1/4}}{D_{h}} \right]^{6} + \left[ 1.63 \Delta T^{1/3} \right]^{6} \right}^{(3x1/6)} + \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [-0.166 + 0.484 \times (ac/h)^{0.8}] \right}^{3}$</td>
</tr>
<tr>
<td>Buoyant</td>
<td>$\left[ 0.6 \frac{(AT)^{1/4}}{D_{h}} \right]^{3} + \left{ \frac{T_{surf} - T_{diffuser}}{\Delta T} \times [-0.166 + 0.484 \times (ac/h)^{0.8}] \right}^{3}$</td>
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*($ac/h$) is ventilation rate measured in room air changes per hour; $\Delta T$ is the absolute value of the surface-air temperature difference (°C); $H$ is the height of vertical surfaces (m); $D_h$ is the hydraulic diameter of horizontal surfaces; $D_h = 4A/P$, where $A$ is the area (m²) and $P$ is the perimeter (m).

Building energy modeling details (example)

- Casual gains
  - Lighting

Developed within IES-VE 5.5.1
Heat transfer through solids

**outside**
- **irradiation**
  - short wave
  - long wave
- reflection

**inside**
- **absorption** (heat source)
- conduction
- convection (forced)
- convection (free)
- emission (long wave)
- exchange (short and long wave)

**storage**
Transient heat conduction

Central to building energy model

- Heat flux reduces in magnitude
- Heat flux shifts in time
- Transient conduction function of:
  - Temperature,
  - Heat flux excitation,
  - Internal heat generation,
  - Temperature-moisture dependent material properties
  - Relative position of individual materials (properties)
Transient heat conduction

- Errors important effect on accuracy total (integrated) building thermal simulation

- Not just for energy prediction and thermal comfort

E.g. control actions based on surface temperature

Condensation:
- Structural damage (Building fabric)
- Mould growth (Health)
Transient heat conduction

• General Fourier heat equation without internal heat source and 1-dimensional

\[
\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2} = a \frac{\partial^2 T}{\partial x^2}
\]
Electrical analogue

example

another example
Numerical approximation of

Finite difference discretization, 1

- *Discretization in space and time*

\[
\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial x^2}
\]

\[
\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot \frac{\partial^2 T}{\partial x^2}
\]
Numerical approximation of 

\[
\frac{\partial T}{\partial t} = a \cdot \frac{\partial^2 T}{\partial x^2}
\]

Based on Taylor series expansion / truncation:

- **Approximation of first order derivatives**

  \[\Delta t = t_{i+1} - t_i \text{ and } \Delta x = x_{i+1} - x_i\]

  \[
  \left. \frac{\partial T}{\partial t} \right|_{x_i,t_{n+1/2}} \approx \frac{T_{i,n+1} - T_{i,n}}{\Delta t}
  \]

  \[
  \left. \frac{\partial T}{\partial x} \right|_{x_{i+1/2},t_n} \approx \frac{T_{i+1,n} - T_{i,n}}{\Delta x}
  \]

- **Approximation of second order derivatives**

  \[
  \left. \frac{\partial^2 T}{\partial x^2} \right|_{x_i,t_n} \approx \frac{\left. \frac{\partial T}{\partial x} \right|_{x_{i+1/2},t_n} - \left. \frac{\partial T}{\partial x} \right|_{x_{i-1/2},t_n}}{\Delta x} \approx \frac{T_{i-1,n} - 2T_{i,n} + T_{i+1,n}}{(\Delta x)^2}
  \]
Numerical approximation of

- Fourier equation (explicit method)

\[
\frac{T(x + \delta x, t) - 2T(x, t) + T(x - \delta x, t)}{(\delta x^2)} = \frac{1}{\alpha} \frac{T(x, t + \delta t) - T(x, t)}{\delta t}
\]

\[
T(x, t + \delta t) = \alpha \frac{\delta t}{(\delta x)^2} T(x - \delta x, t) + \left(1 - 2\alpha \frac{\delta t}{(\delta x)^2}\right) T(x, t) + \alpha \frac{\delta t}{(\delta x)^2} T(x + \delta x, t)
\]

Explicit schemes:
Easy to solve
Unstable under circumstances

Stability criterion
\[
\frac{\alpha \delta t}{(\delta x)^2} \leq \frac{1}{2}; \quad F \leq \frac{1}{2}
\]

F = Fourier number
Numerical approximation of

- Fourier equation (implicit method)

\[ \frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial x^2} \]

\[ \frac{T(x + \delta x, t + \delta t) - 2T(x, t + \delta t) + T(x, t)}{(\delta x)^2} = \frac{1}{\alpha} \frac{T(x, t + \delta t) - T(x, t)}{\delta t} \]

\[ \left(1 + 2\alpha \frac{\delta t}{(\delta x)^2}\right)T(x, t + \delta t) = T(x, t) + \alpha \frac{\delta t}{(\delta x)^2} \left[T(x + \delta x, t + \delta t) + T(x - \delta x, t + \delta t)\right] \]

Implicit schemes:
- Unconditionally stable
- Simultaneous solution of all connected equations
- Large \( \delta x, \delta t > \) discretisation errors
Numerical approximation of

- Fourier equation (general)

\[
(1 + 2\gamma F)T(x, t + \delta t) = \gamma F[T(x + \delta x, t + \delta t) + T(x - \delta x, t + \delta t)] \\
+ [1 - 2F(1 - \gamma)]T(x, t) + (1 - \gamma)F[T(x + \delta x, t) + T(x - \delta x, t)]
\]

\(\gamma = 0\) explicit scheme  
\(\gamma = 1\) full implicit scheme  
\(\gamma = 0.5\) Crank-Nicolson
Errors (= difference exact result - numerical approximation) are due to:

- Truncation errors
- Rounding errors
- “History” errors

History error

- Errors from preceding solution steps
- Dependent on growth factor ($\Lambda$)

$\Lambda > 1$: unbounded growth
$|\Lambda| < 1$: stable solution
$\Lambda < 0$: oscillation may occur
$\Lambda < -1$: unstable oscillation

- Implicit schemes ($\gamma = 0.5-1$): $|\Lambda| < 1$
  - $F = 1/4(1-\gamma)$: $\Lambda = 0$
  - $F > 1/4(1-\gamma)$: oscillation may occur
Transient heat conduction

- Accuracy versus computational effort.
Transient heat conduction

- Distribution of nodes
  - Heat wave fluctuations
  - Effective thickness ($d^*$)
  - Node within region $d^*$

$$d^* = \sqrt{\alpha t_0 / \pi}$$

<table>
<thead>
<tr>
<th>Cycle time ($t_0$)</th>
<th>$d^*$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>for concrete ($\alpha = 10^{-6} \text{ m}^2\text{s}^{-1}$)</td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>3.17</td>
</tr>
<tr>
<td>1 day</td>
<td>0.17</td>
</tr>
<tr>
<td>1 hour</td>
<td>0.034</td>
</tr>
<tr>
<td>1 minute</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Simulation example

2mm aluminium (1 layer), surface temperature

- **ambient**
- **exact**
- timestep 3600 s, F = 292208
- timestep 1800 s, F = 146104
- timestep 900 s, F = 73052
Simulation example
Simulation example

- Ambient
- Exact
- Time step 3600 s, \( F = 0.35 \)
- Time step 1800 s, \( F = 0.18 \)
- Time step 900 s, \( F = 0.09 \)
Simulation example

200 mm concrete (timestep 80 s), surface temperature

- ambient
- exact
- 1 layer, F = 0.005
- 3 layers, F = 0.042
- 8 layers, F = 0.29
Simulation example

- Errors in surface temperatures > errors in heat flow

Stored heat [kJ/m²] (ΔT = 20K)

<table>
<thead>
<tr>
<th>Description</th>
<th>Aluminium</th>
<th>Insulation</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>49.3</td>
<td>42.8</td>
<td>3984</td>
</tr>
<tr>
<td>Δt = 3600 s</td>
<td>39.8</td>
<td>44.1</td>
<td>3865</td>
</tr>
<tr>
<td>Δt = 1800 s</td>
<td>38.0</td>
<td>43.0</td>
<td>3864</td>
</tr>
<tr>
<td>Δt = 900 s</td>
<td>42.0</td>
<td>42.9</td>
<td>3864</td>
</tr>
</tbody>
</table>
Transient conduction modeling

- **Conclusions**
  - $\gamma > 0.5$ preferable
  - Decreasing time step does not always improve accuracy.
  - Determine “period time” (first node)
  - Time step small enough for detail in time period:
    - 1 h for daily fluctuation
    - 2 min for 10 min steps
  - $\gamma$-$F$ combinations such that growth factor is low
  - Note that above aspects are not always under control of the user! (depends on the program used)